

GCE

Further Mathematics B MEI

Y420/01: Core Pure

A Level

Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

YOU MUST MARK 5 PRACTICE AND 10 STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.
BP	Blank Page
Seen	
Highlighting	

6. Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- If a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.

If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

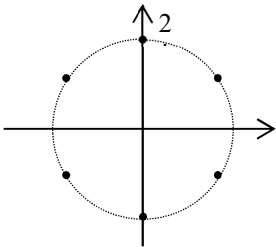
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)	$a - ib$	B1 [1]	1.2	
1	(a)	(ii)	$iz = -b + ai$ so $\text{Re}(iz) = -b$	M1 A1 [2]	1.1 1.1	
1	(b)	(i)	DR $\frac{5 + \sqrt{3}i}{2 - \sqrt{3}i} = \frac{(5 + \sqrt{3}i)(2 + \sqrt{3}i)}{(2 - \sqrt{3}i)(2 + \sqrt{3}i)}$ $= \frac{7 + 7\sqrt{3}i}{7}$ $= 1 + \sqrt{3}i$	M1 A1 [2]	1.1a 1.1	an intermediate step must be seen before final answer
1	(b)	(ii)	$ 1 + \sqrt{3}i = 2$ $\arg(1 + \sqrt{3}i) = \frac{\pi}{3}$ so $w = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$	B1ft B1 [2]	1.1 1.1	for either their modulus or argument soi cao. Allow 60°

Question			Answer	Marks	AO	Guidance
2			DR Normal to plane is $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Angle between $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is θ $\cos \theta = \frac{3 \times (-1) + 2 \times 3 + 1 \times 2}{\sqrt{3^2 + 2^2 + 1^2} \sqrt{(-1)^2 + 3^2 + 2^2}} = \frac{5}{\sqrt{14}\sqrt{14}}$ $\Rightarrow \theta = 69.07\dots$ 90 – their θ so angle with plane is 20.9°	B1 M1 A1 M1 A1	1.1a 1.1 1.1 1.1 2.2a	soi use of scalar product formula with their normal seen or $1.2\dots$ rads or $\arccos \frac{5}{14}$ or $\frac{\pi}{2} - \text{their } \theta$ or 0.365 rads
			Alternative method 1 Normal to plane is $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Angle between $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the plane is θ $\sin \theta = \frac{3 \times (-1) + 2 \times 3 + 1 \times 2}{\sqrt{3^2 + 2^2 + 1^2} \sqrt{(-1)^2 + 3^2 + 2^2}} = \frac{5}{\sqrt{14}\sqrt{14}}$ $\theta = 20.9^\circ$	B1 M2 A2		soi or 0.365 rads
			Alternative method 2 Normal to plane is $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Angle between $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is θ $\sin \theta = \frac{\left \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \right }{\sqrt{3^2 + 2^2 + 1^2} \sqrt{(-1)^2 + 3^2 + 2^2}} = \frac{\left \begin{pmatrix} 1 \\ -7 \\ 11 \end{pmatrix} \right }{\sqrt{14}\sqrt{14}}$ $\Rightarrow \theta = 69.07\dots$ 90 – their θ so angle with plane is 20.9°	B1 M1 A1 M1 A1		soi complete method with vector product seen or $1.2\dots$ rads or $\arcsin \frac{\sqrt{171}}{14}$ or $\frac{\pi}{2} - \text{their } \theta$ or 0.365 rads
				[5]		

Question			Answer	Marks	AO	Guidance
3	(a)		$\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{(r+2)}$ $1 = A(r+2) + Br$ $r = 0 \Rightarrow A = \frac{1}{2}, \quad r = -2 \Rightarrow B = -\frac{1}{2}$ $\sum_{r=1}^n \frac{1}{r(r+2)} = \frac{1}{2} \sum_{r=1}^n \left(\frac{1}{r} - \frac{1}{r+2} \right)$ $= k \left[1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{n} - \frac{1}{n+2} \right]$ $= \frac{1}{2} \left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right]$ $= \frac{3}{4} - \frac{1}{2(n+1)(n+2)}$	M1 A1 M1 A1 A1 [5]	1.1 1.1 1.1 1.1 1.1	 enough terms to show consistent cancellation in their series
3	(b)		$\frac{3}{4}$	B1 [1]	2.2a	

Question			Answer	Marks	AO	Guidance
4	(a)	(i)	$f'(x) = (1 + 2x)^{-\frac{1}{2}}$ $f''(x) = -(1 + 2x)^{-\frac{3}{2}}$	B1	1.1	
				B1 [2]	1.1	
4	(a)	(ii)	$f(0) = 1, f'(0) = 1, f''(0) = -1$ $1 + x - \frac{1}{2}x^2$	M1 A1 [2]	1.1 1.1	Their $f(0)$, $f'(0)$ and $f''(0)$ evaluated and substituted into Maclaurin Must come from correct expressions for $f'(x)$, $f''(x)$; cannot come from binomial expansion. Ignore subsequent terms.
4	(b)		$\sqrt{1 + 2 \times \frac{1}{8}}$ or $1 + \frac{1}{8} - \frac{1}{2}\left(\frac{1}{8}\right)^2$ $\Rightarrow \sqrt{5} \approx \frac{143}{64}$	M1 A1 [2]	3.1a 2.2a	using $x = \frac{1}{8}$ in their expansion AG

Question			Answer	Marks	AO	Guidance
5	(a)		DR Let $z = r(\cos \theta + i \sin \theta)$ or $z = re^{i\theta}$ $z^6 = 64(\cos \pi + i \sin \pi)$ or $64e^{i\pi}$ $\Rightarrow r = 2$ $z = 2e^{i\pi/6}, 2e^{i\pi/2}, 2e^{5i\pi/6}, 2e^{-i\pi/6}, 2e^{-i\pi/2}, 2e^{-5i\pi/6}$	M1 B1 A1 A1 [4]	1.1 1.1 2.5 1.1	condone $-\pi$ for π soi 1 correct all correct or $2e^{7i\pi/6}, 2e^{3i\pi/2}, 2e^{11i\pi/6}$
5	(b)			M1 A1 A1 [3]	1.1 1.1 1.1	six roots lying on approximate circle centre O forming an approximate regular hexagon root at $2i$ or $-2i$ oe indicated. If part (a) contains incorrect roots do not award final A1.

Question			Answer	Marks	AO	Guidance
6	(a)		DR $MN = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ $NM = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ or $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ so not commutative	M1 A1* A1 [3]	1.1a 1.1 2.2a	both MN and NM calculated both correct dep A1*
6	(b)	(i)	M is reflection in $y = x$	B1 [1]	1.1	
6	(b)	(ii)	N is stretch parallel to the x -axis scale factor 2	M1 A1 [2]	1.1 1.1	
6	(c)		order of transformations matters	B1 [1]	2.2a	must refer to transformations not just matrices
6	(d)		$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ y \end{pmatrix}$ or $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} 2x \\ mx + c \end{pmatrix}$ y -coordinate unchanged so lines parallel to x -axis invariant	M1 A1 [2]	2.1 2.2a	Allow SC1 for correct geometrical argument, e.g. stretch in x -direction leaves y -coordinates unchanged www. Must refer to invariant lines.

Question			Answer	Marks	AO	Guidance
7	(a)		A: $r = a$, $\theta = 0$	B1	1.1	or $\theta = 2\pi$
			B: $r = -a$, $\theta = \pi/2$	B1	1.1	
			C: $r = 3a$, $\theta = 3\pi/2$	B1	1.1	
				[3]		
7	(b)		$\sin \theta > \frac{1}{2}$	M1	3.1a	Allow $\sin \theta = \frac{1}{2}$ (or $< \frac{1}{2}$ or $\leq \frac{1}{2}$ or $\geq \frac{1}{2}$)
			$\frac{1}{6}\pi, \frac{5}{6}\pi$	A1	1.1	
			$\frac{1}{6}\pi < \theta < \frac{5}{6}\pi$	A1	1.1	
				[3]		

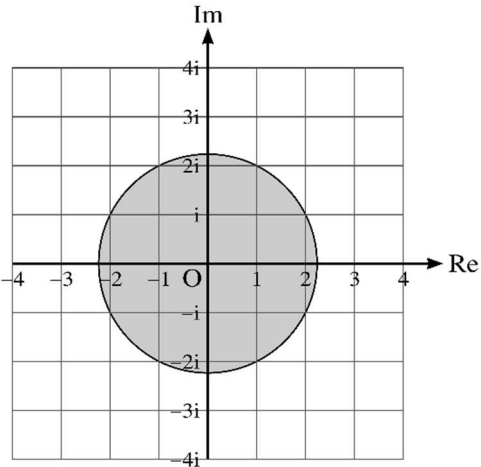
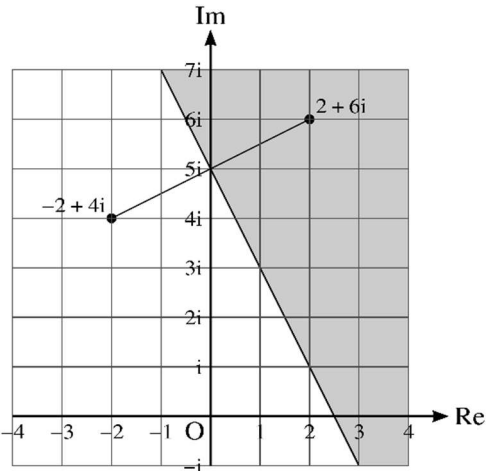
Question			Answer	Marks	AO	Guidance
8			when $n = 1$, $8^n - 3^n = 5$ div by 5 [Assume true when $n = k$] so $8^k = 3^k + 5m$ $8^{k+1} - 3^{k+1} = 8(3^k + 5m) - 3 \times 3^k$ $= 5 \times 3^k + 40m = 5(3^k + 8m)$ div by 5	B1 M1 M1 A1	2.1 2.1 2.1 2.2a	or $3^k = 8^k - 5m$ or $8^k - 3^k = 5m$ or $8^k - 3^k$ div 5 assumption used successful completion $3^k = 8^k - 5m$ used $\Rightarrow 5(8^k + 3m)$ div 5 www $8^k - 3^k = 5m$ used $\Rightarrow 5(8m + 3^k)$ div 5 www
			so if true for $n = k$ then true for $n = k+1$ As true for $n = 1$, true for all n .	A1	2.4	must receive all previous marks for this to be awarded
			Alternative method when $n = 1$, $8^n - 3^n = 5$ div by 5 [Assume true when $n = k$] so $u_k = 8^k - 3^k = 5m$ $u_{k+1} - u_k = 8^{k+1} - 3^{k+1} - (8^k - 3^k)$ $= 2(8^k - 3^k) + 5(8^k)$ $= 2(5m) + 5(8^k) = 5(2m + 8^k)$ div by 5	B1 M1 M1 A1		considering difference between u_{k+1} and u_k
			so if true for $n = k$ then true for $n = k+1$ As true for $n = 1$, true for all n .	A1		must receive all previous marks for this to be awarded
				[5]		

Question			Answer	Marks	AO	Guidance		
9			$k \int_0^{\frac{2\pi}{n}} \sin^2 nt \, dt = \frac{1}{2} k \int_0^{\frac{2\pi}{n}} (1 - \cos 2nt) dt$	M1*	3.1a	use of double angle formula for $\sin^2 nt$		
				A1	1.1			
			$= \frac{1}{2} k \left[t - \frac{1}{2n} \sin 2nt \right]_0^{\frac{2\pi}{n}}$	M1dep	1.1	$\left[t - \frac{1}{2n} \sin 2nt \right]$		
			$= \frac{1}{2} a^2 \frac{2\pi}{n} = \frac{\pi a^2}{n}$	A1	1.1	www		
			mean value of $I^2 = \frac{\pi a^2}{n} \bigg/ \frac{2\pi}{n} = \frac{a^2}{2}$	M1	1.1	for dividing by $\frac{2\pi}{n}$		
			$\Rightarrow$ RMS value $= \frac{a}{\sqrt{2}}$	A1	2.2a	www AG		
			Alternative method					
			$\frac{1}{\left(\frac{2\pi}{n}\right)} k \int_0^{\frac{2\pi}{n}} \sin^2 nt \, dt = \frac{n}{2\pi} k \int_0^{\frac{2\pi}{n}} \sin^2 nt \, dt$	M1	1.1	for dividing their integral by $\frac{2\pi}{n}$		
			$= \frac{1}{2} \times \frac{n}{2\pi} k \int_0^{\frac{2\pi}{n}} (1 - \cos 2nt) dt$	M1*	3.1a	use of double angle formula for $\sin^2 nt$		
				A1	1.1			
			$= \frac{n}{4\pi} k \left[t - \frac{1}{2n} \sin 2nt \right]_0^{\frac{2\pi}{n}}$	M1dep	1.1	$\left[t - \frac{1}{2n} \sin 2nt \right]$		
			$= \frac{a^2 n}{4\pi} \left(\frac{2\pi}{n} \right) = \frac{a^2}{2}$	A1	1.1	www		
$\Rightarrow$ RMS value $= \frac{a}{\sqrt{2}}$	A1	2.2a	www AG					
				[6]				

Question			Answer	Marks	AO	Guidance
10	(a)		$\alpha + \beta + \alpha + \beta = 4 \quad [\Rightarrow \alpha + \beta = 2]$	B1	3.1a	or $\alpha\beta + 2\alpha + 2\beta = 7$ or $\alpha\beta + 2(\alpha + \beta) = 7$ substitution which could lead to a quadratic in α or β or x could be in α or β a method to solve their quadratic
			$\alpha\beta + (\alpha + \beta)\alpha + (\alpha + \beta)\beta = 7$	B1	1.1	
			$\Rightarrow \alpha^2 + \beta^2 + 3\alpha\beta = 7$			
			$x + \frac{3}{x} = 2$ or $3x(2 - x) + (2 - x)^2 + x^2 = 7$	M1	3.1a	
			$\Rightarrow x^2 - 2x + 3 = 0$	A1	1.1	
			$\Rightarrow x = \frac{2 \pm \sqrt{-8}}{2} = 1 \pm i\sqrt{2}$	M1	1.1	
10	(b)		so roots are $1 + i\sqrt{2}$, $1 - i\sqrt{2}$ and 2	A1	2.2a	
				[6]		
10	(b)		$[c =] - 6$	B1	2.2a	
				[1]		

Question			Answer	Marks	AO	Guidance
11			$\frac{dy}{dx} - 2y \frac{\sinh x}{\cosh x} = 1$	B1	3.1a	$\frac{dy}{dx} - 2y \tanh x = 1$
			IF is $e^{\int -\frac{2 \sinh x}{\cosh x} dx}$	M1	2.1	or $e^{\int -2 \tanh x \, dx}$. Integral must come from an attempt to get $\frac{dy}{dx}$ on its own
			$= e^{-2 \ln \cosh x} = (e^{\ln \operatorname{sech} x})^2 = \operatorname{sech}^2 x$	A1	2.2a	
			$\frac{d}{dx}(y \operatorname{sech}^2 x) = \operatorname{sech}^2 x$	M1	2.1	or multiplying through by their IF
			$y \operatorname{sech}^2 x = \int \operatorname{sech}^2 x \, dx + c = \tanh x + c$	A1	1.1	
			substituting $x = 0$, $y = 1$ gives $c = 1$	M1	1.1	substituting $x = 1$ and $y = 1$ to lead to a value for c
			$y = \cosh^2 x (\tanh x + 1)$	A1	2.2a	oe, e.g. $\cosh x (\sinh x + \cosh x)$ or $e^x \cosh x$
				[7]		

Question			Answer	Marks	AO	Guidance
12			Considering $\left(z - \frac{1}{z}\right)^5$	M1	3.1a	
			$32i \sin^5 \theta$	B1	1.1	seen at any stage
			$= z^5 - 5z^3 + 10z - 10\frac{1}{z} + 5\frac{1}{z^3} - \frac{1}{z^5}$	M1	1.1	expansion of $\left(z - \frac{1}{z}\right)^5$
			$= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$	A1	2.1	correct expansion with $z^n, \frac{1}{z^n}$ terms paired and factorised
			Using $z^n - \frac{1}{z^n} = 2i \sin n\theta$ with sum of terms	M1	2.1	FT their expansion
			$= 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta$	A1	1.1	i must appear in each term
			$\Rightarrow \sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$	A1	2.2a	www
			Alternative method 1			
			Considering $(e^{i\theta} - e^{-i\theta})^5$	M1		
			$32i \sin^5 \theta$	B1		
			$= e^{5i\theta} - 5e^{3i\theta} + 10e^{i\theta} - 10e^{-i\theta} + 5e^{-3i\theta} - e^{-5i\theta}$	M1		expansion of $(e^{i\theta} - e^{-i\theta})^5$
			$= e^{5i\theta} - e^{-5i\theta} - 5(e^{3i\theta} - e^{-3i\theta}) + 10(e^{i\theta} - e^{-i\theta})$	A1		correct expansion with $e^{i\theta}, e^{-i\theta}$ terms paired and factorised
Using $e^{i\theta} - e^{-i\theta} = 2i \sin n\theta$ with sum of terms	M1		FT their expansion			
$= 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta$	A1		i must appear in each term			
$\Rightarrow \sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$	A1		www			
Alternative method 2						
Equating Im components of $(\cos \theta + i \sin \theta)^5$			using binomial expansion and de Moivre's theorem			
$\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$	B1					
$= 5(1 - \sin^2 \theta)^2 \sin \theta - 10(1 - \sin^2 \theta) \sin^3 \theta + \sin^5 \theta$	M1*		correct expression for $\sin 5\theta$ and substituting in $\cos^2 \theta = 1 - \sin^2 \theta$			
$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$	A1		or $16 \sin^5 \theta = \sin 5\theta + 20 \sin^3 \theta - 5 \sin \theta$ or any correct rearrangement			
Equating Im components of $(\cos \theta + i \sin \theta)^3$			using binomial expansion and de Moivre's theorem			
$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$	M1*		correct expression for $\sin 3\theta$ in terms of $\sin \theta$ and $\cos \theta$			
$\sin^3 \theta = \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta$	A1		or $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ or any correct rearrangement			
$\Rightarrow 16 \sin^5 \theta = \sin 5\theta + 20\left(\frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta\right) - 5 \sin \theta$	M1dep		substituting expression for $\sin^3 \theta$ into $\sin^5 \theta$.			
$\Rightarrow \sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$	A1		www			

Question			Answer	Marks	AO	Guidance
13	(a)	(i)		M1 A1 A1	1.1 1.1 1.1	circle, centre O radius $\sqrt{5}$ shaded inside (oe). Candidates may shade the region that is not required, but should clearly indicate that what they have shaded is not required.
13	(a)	(ii)		M1 A1 A1	1.1 1.1 1.1	$(-2, 4)$ and $(2, 6)$ identified perpendicular bisector of $(-2, 4)$ and $(2, 6)$ shaded on RHS of line (oe). Candidates may shade the region that is not required, but should clearly indicate that what they have shaded is not required.

Question			Answer	Marks	AO	Guidance
13	(b)		gradient = -2 passing through $(0, 5)$	M1 B1	1.1 3.1a	
			equation $y = -2x + 5$	A1	1.1	oe. Allow inequality. Could be obtained from diagram in (a).
			circle is $x^2 + y^2 = 5$ $x^2 + (5 - 2x)^2 = 5$	B1 M1	3.1a 2.1	allow inequality or $\left(\frac{5}{2} - \frac{y}{2}\right)^2 + y^2 = 5$. Must be an equation.
			$\Rightarrow 5x^2 - 20x + 20 = 0$ $\Rightarrow x = 2$ [only] [unique solution is] $z = 2 + i$	M1 A1* A1dep	1.1 2.2a 3.2a	simplifying to a three-term quadratic equation or $y = 1$ [only]
			Alternative method $(x + 2)^2 + (y - 4)^2 = (x - 2)^2 + (y - 6)^2$	M1 M1 A1 B1 M1		squaring both sides of equations or inequality expanding all four sets of brackets oe. Allow inequality. allow inequality
			$y = -2x + 5$ circle is $x^2 + y^2 = 5$ $x^2 + (5 - 2x)^2 = 5$			or $\left(\frac{5}{2} - \frac{y}{2}\right)^2 + y^2 = 5$. Must be an equation.
			$\Rightarrow 5x^2 - 20x + 20 = 0$ $\Rightarrow x = 2$ [only] [unique solution is] $z = 2 + i$	M1 A1* A1dep		simplifying to a three term quadratic equation or $y = 1$ [only]
				[8]		

Question			Answer	Marks	AO	Guidance
14	(a)		$\begin{vmatrix} k & 0 & -1 \\ -1 & k & 2 \\ 2k & 2 & 3 \end{vmatrix} = k(3k - 4) - 1(-2 - 2k^2)$ $= 5k^2 - 4k + 2$ discriminant = $16 - 40 = -24 < 0$ so determinant is never zero $\Rightarrow$ planes always meet at a point	M1 M1 A1 M1 A1 [5]	1.1 1.1 1.1 3.1a 2.2a	considering correct determinant finding determinant using any row or column oe, e.g. completing the square, considering quadratic formula or showing no real roots www. Both statements required.
14	(b)		$\mathbf{M}^{-1} = \frac{1}{5k^2 - 4k + 2} \begin{pmatrix} 3k - 4 & -2 & k \\ 4k + 3 & 5k & -2k + 1 \\ -2 - 2k^2 & -2k & k^2 \end{pmatrix}$ $\mathbf{M}^{-1} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{5k^2 - 4k + 2} \begin{pmatrix} 6k - 10 \\ 13k + 6 \\ -4k^2 - 2k - 4 \end{pmatrix}$ $\left(\frac{6k - 10}{5k^2 - 4k + 2}, \frac{13k + 6}{5k^2 - 4k + 2}, \frac{-4k^2 - 2k - 4}{5k^2 - 4k + 2} \right)$	M1 A1 M1 M1 A1 M1 A2,1,0 [8]	1.1 1.1 1.1 3.1a 1.1 1.1 1.1, 1.1	at least 5 cofactors correct (need not be in matrix) all cofactors correct cofactor matrix transposed Multiplying by $\frac{1}{\text{their determinant}}$ correct inverse finding $\mathbf{M}^{-1} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ Award A1 for one of x, y or z coordinates correct or all three correct FT their determinant

Question			Answer	Marks	AO	Guidance
15			DR $1 + 2x - x^2 = -(x^2 - 2x) + 1 = -(x - 1)^2 + 2$	M1	3.1a	completing the square
			$\left[\arcsin \frac{x-1}{k} \right]_1^2$	A1	1.1	
			$= \left[\arcsin \frac{x-1}{\sqrt{2}} \right]_1^2$	M1	2.1	
			$= \arcsin \left(\frac{1}{\sqrt{2}} \right) - \arcsin 0 = \frac{\pi}{4}$	A1	2.1	substitution or evaluation of both limits must be seen
			Alternative method $1 + 2x - x^2 = -(x^2 - 2x) + 1 = -(x - 1)^2 + 2$	A1	2.2a	
			Let $u = x - 1$ $\int_0^1 \frac{1}{\sqrt{2-u^2}} du$	M1		completing the square
			$= \left[\arcsin \frac{u}{\sqrt{2}} \right]_0^1$	A1		complete substitution including limits
			$= \arcsin \left(\frac{1}{\sqrt{2}} \right) - \arcsin 0 = \frac{\pi}{4}$	A1		substitution or evaluation of both limits must be seen
				[5]		

Question	Answer	Marks	AO	Guidance
16	Distance from point to plane = $\frac{ 2 \times 4 + 1 \times 1 + 0 \times 2 }{\sqrt{2^2 + 1^2 + 2^2}}$ = 3 units Line is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + b\mathbf{j} + 3\mathbf{k})$ $\overrightarrow{AP} = \mathbf{i} + 5\mathbf{k}$ $\overrightarrow{AP} \times \mathbf{d} = (\mathbf{i} + 5\mathbf{k}) \times (2\mathbf{i} + b\mathbf{j} + 3\mathbf{k})$ $= -5b\mathbf{i} + 7\mathbf{j} + b\mathbf{k}$ dist from P to line $\frac{ -5b\mathbf{i} + 7\mathbf{j} + b\mathbf{k} }{ \mathbf{d} } = \frac{\sqrt{26b^2 + 49}}{\sqrt{13 + b^2}}$ so $\frac{26b^2 + 49}{13 + b^2} = 9 \Rightarrow 26b^2 + 49 = 117 + 9b^2$ $\Rightarrow b = 2$	M1* A1 M1 A1 M1 A1 M1* A1 M1dep A1	3.1a 1.1 3.1a 2.1 2.1 2.1 3.1a 1.1 2.1 3.2a	 or $-\overrightarrow{AP}$. Could be seen as part of distance calculation. or $\mathbf{d} \times \overrightarrow{AP}$ equating the two distances
	Alternative method 1 Distance from point to plane = $\frac{ 2 \times 4 + 1 \times 1 + 0 \times 2 }{\sqrt{2^2 + 1^2 + 2^2}}$ = 3 units Line is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + b\mathbf{j} + 3\mathbf{k})$ $(2\lambda - 1)\mathbf{i} + b\lambda\mathbf{j} + (3\lambda - 5)\mathbf{k}$ $\begin{pmatrix} 2\lambda - 1 \\ b\lambda \\ 3\lambda - 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ b \\ 3 \end{pmatrix} = 0$ $\lambda = \frac{17}{13 + b^2}$ dist from P to line $\sqrt{\left(2\left(\frac{17}{13 + b^2}\right) - 1\right)^2 + \left(b\left(\frac{17}{13 + b^2}\right)\right)^2 + \left(3\left(\frac{17}{13 + b^2}\right) - 5\right)^2}$ $= \frac{\sqrt{26b^4 + 387b^2 + 637}}{13 + b^2}$ So $\frac{26b^4 + 387b^2 + 637}{(13 + b^2)^2} = 9$ $\Rightarrow 17b^4 + 153b^2 - 884 = 0$ $\Rightarrow b = 2$	M1* A1 M1 A1 M1* A1 M1 A1 M1dep A1		 vector from P to a point on the line substituting λ into $ \overrightarrow{AP} $ simplified equating the two distances

Question	Answer	Marks	AO	Guidance
	Alternative method 2 Distance from point to plane = $\frac{ 2 \times 4 + 1 \times 1 + 0 \times 2 }{\sqrt{2^2 + 1^2 + 2^2}}$ = 3 units Line is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + b\mathbf{j} + 3\mathbf{k})$ $(2\lambda - 1)\mathbf{i} + b\lambda\mathbf{j} + (3\lambda - 5)\mathbf{k}$ $\sqrt{(2\lambda - 1)^2 + (b\lambda)^2 + (3\lambda - 5)^2}$ $\sqrt{(13 + b^2)\lambda^2 - 34\lambda + 26}$ $\sqrt{(13 + b^2)\lambda^2 - 34\lambda + 26} = 3$ $(13 + b^2)\lambda^2 - 34\lambda + 17 = 0$ So $(-34)^2 - 4(13 + b^2)(17) = 0$ $\Rightarrow b = 2$ Alternative method 3 Distance from point to plane = $\frac{ 2 \times 4 + 1 \times 1 + 0 \times 2 }{\sqrt{2^2 + 1^2 + 2^2}}$ = 3 units Line is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + b\mathbf{j} + 3\mathbf{k})$ $\overrightarrow{AP} = \mathbf{i} + 5\mathbf{k}$ $\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ b \\ 3 \end{pmatrix} = \left \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} \right \left \begin{pmatrix} 2 \\ b \\ 3 \end{pmatrix} \right \cos \theta$ $1 \times 2 + 5 \times 3 = \sqrt{1^2 + 5^2} \sqrt{2^2 + b^2 + 3^2} \cos \theta$ $\cos \theta = \frac{17}{\sqrt{26} \sqrt{13 + b^2}}$ $\sqrt{26} \cos \theta = \frac{17}{\sqrt{13 + b^2}}$ $26 - \left(\frac{17}{\sqrt{13 + b^2}} \right)^2 = 3^2$ $\Rightarrow b = 2$	M1* A1 M1 A1 M1* A1 M1dep A1 M1 A1 M1* M1 A1 M1* M1dep A1 [10]		vector from P to a point on the line finding magnitude of this vector expanding and simplifying setting distances equal correct quadratic equation = 0. setting discriminant equal to 0 or $-\overrightarrow{AP}$. Could be seen as part of distance calculation. scalar product formula with $\overrightarrow{AP} \cdot \mathbf{d}$ to find a value for $\cos \theta$ evaluating scalar product and magnitudes expression for $\cos \theta$ expression for the distance from (3, 1, -5) to the foot of the perpendicular to the line using their value correctly with Pythagoras oe to lead to a value for b

Question			Answer	Marks	AO	Guidance
17	(a)	(i)	$\frac{d^2x}{dt^2} = k \frac{dx}{dt} - a \frac{dy}{dt}$ $= k \frac{dx}{dt} - a(ky - bx)$ $= k \frac{dx}{dt} + abx - k \left(kx - \frac{dx}{dt} \right)$ $\frac{d^2x}{dt^2} - 2k \frac{dx}{dt} + (k^2 - n^2)x = 0$ $\text{AE } m^2 - 2km + (k^2 - n^2) = 0 \Rightarrow m = k \pm n$ $\text{Hence GS is } x = Ae^{(k+n)t} + Be^{(k-n)t}$	M1*	3.1a	differentiate $\frac{dx}{dt}$ wrt t
				M1dep	1.1	substitution for $\frac{dy}{dt}$
				M1dep	3.1a	substitution for y
				A1	2.1	correct differential equation = 0. Could see ab instead of n^2 .
				M1	2.1	giving and solving their AE
				A1	2.3	AG www
			Alternative method $y = \frac{k}{a}x - \frac{1}{a} \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{k}{a} \frac{dx}{dt} - \frac{1}{a} \frac{d^2x}{dt^2}$ $\frac{k}{a} \frac{dx}{dt} - \frac{1}{a} \frac{d^2x}{dt^2} = ky - bx$ $\frac{k}{a} \frac{dx}{dt} - \frac{1}{a} \frac{d^2x}{dt^2} = k \left(\frac{k}{a}x - \frac{1}{a} \frac{dx}{dt} \right) - bx$ $\frac{d^2x}{dt^2} - 2k \frac{dx}{dt} + (k^2 - n^2)x = 0$ $\text{AE } m^2 - 2km + (k^2 - n^2) = 0 \Rightarrow m = k \pm n$ $\text{Hence GS is } x = Ae^{(k+n)t} + Be^{(k-n)t}$	M1*		differentiate y wrt t
				M1dep		substitution for $\frac{dy}{dt}$
				M1dep		substitution for y
				A1		correct differential equation = 0. Could see ab instead of n^2 .
				M1		giving and solving their AE
				A1		AG www
				[6]		
17	(a)	(ii)	$\frac{dx}{dt} = A(k+n)e^{(k+n)t} + B(k-n)e^{(k-n)t}$ $y = \frac{n}{a}(-Ae^{(k+n)t} + Be^{(k-n)t})$	M1	2.1	differentiation of x must be seen
				A1	2.2a	may not be factorised but must be simplified
				[2]		

Question			Answer	Marks	AO	Guidance
17	(b)	(i)	$n = \sqrt{0.004} = 0.02 \Rightarrow k + n = 0.035 \text{ and } k - n = -0.005$	M1	1.1	all three values established
			$x_0 = A + B, \quad y_0 = -\frac{1}{2}(A - B)$	M1	3.3	a method to find both A and B (ft their y) from expressions for x_0 and y_0
			$\Rightarrow A = \frac{1}{2}(x_0 - 2y_0), \quad B = \frac{1}{2}(x_0 + 2y_0)$	A1	2.2a	AG dependent upon both M1s
			$x = \frac{1}{2}(x_0 - 2y_0)e^{0.035t} + \frac{1}{2}(x_0 + 2y_0)e^{-0.005t}$			
17	(b)	(ii)	Alternative method $n = \sqrt{0.004} = 0.02 \Rightarrow k + n = 0.035, k - n = -0.005$	M1		all three values established
			$\frac{dx}{dt} = 0.015x_0 - 0.04y_0, \quad \frac{dx}{dt} = 0.035A - 0.005B$	M1		a method to find both A and B from two expressions for $\frac{dx}{dt}$
			$\Rightarrow A = \frac{1}{2}(x_0 - 2y_0), \quad B = \frac{1}{2}(x_0 + 2y_0)$	A1		AG dependent upon both M1s
			$x = \frac{1}{2}(x_0 - 2y_0)e^{0.035t} + \frac{1}{2}(x_0 + 2y_0)e^{-0.005t}$	[3]		
17	(b)	(ii)	$y = \frac{1}{2}\left(-\frac{1}{2}(x_0 - 2y_0)e^{0.035t} + \frac{1}{2}(x_0 + 2y_0)e^{-0.005t}\right)$ $= \frac{1}{4}(x_0 + 2y_0)e^{-0.005t} - \frac{1}{4}(x_0 - 2y_0)e^{0.035t}$	B1 [1]	2.2a	AG first step or equivalent substitution must be seen
17	(c)	(i)	$x = -50e^{0.875} + 550e^{-0.125},$ $y = 275e^{-0.125} + 25e^{0.875}$	M1	1.1	for either may be implied by awrt 365 or 303
			$x = 365, y = 302$	A1 [2]	2.2b	for both correct allow $y = 303$

Question			Answer	Marks	AO	Guidance
17	(c)	(ii)	DR $-50e^{0.035t} + 550e^{-0.005t} = 275e^{-0.005t} + 25e^{0.035t}$ $275e^{-0.005t} = 75e^{0.035t}$ $0.04t = \ln\left(\frac{11}{3}\right)$ $t = 32.5$, so numbers equal after about 32 or 33 years	M1 M1 M1 A1 [4]	3.1b 2.1 3.1a 2.2a	equating x and y with x_0, y_0 substituted collecting like terms oe. Taking logs for a single term in t .
17	(c)	(iii)	x does become zero, as $550e^{-0.005t} \rightarrow 0$ as t increases, and $-50e^{0.035t}$ is always negative y is the sum of two positive terms, so is never zero	M1 A1 B1 [3]	3.4 3.4 3.5a	M1 for correct conclusion with some explanation A1 for complete explanation (e.g. $x = 0$ at $t = 59.94\dots$) reason must be given, e.g. $y = 0$ when $t = 25 \ln(-11)$ which is undefined or $e^{0.04t} = -11$ has no solution. Implied by M1A1 without further working seen unless incorrect conclusion for y .
17	(d)	(i)	$x_0 = 2y_0$	B1 [1]	3.5b	oe. Subscripts must be seen.
17	(d)	(ii)	$x = Ce^{-0.005t}, y = \frac{1}{2}Ce^{-0.005t}$ so both population numbers tend to zero	M1 A1 [2]	3.3 2.4	oe; e.g. C is $\frac{1}{2}(x_0 + 2y_0)$ or $2y_0$ or x_0 oe, e.g. both species disappear. Must be supported by explanation.

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